

A Mass Inequality for the Ξ^* and Θ^+ Pentaquarks

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Abstract

We derive an upper bound on the mass difference between the Ξ^* and Θ^+ pentaquarks which are the manifestly exotic members of the $SU(3)_f$ antidecuplet. The derivation is based on simple assumptions about $SU(3)_f$ symmetry breaking and uses the standard quantum mechanical variational method. The resulting rather robust bound is more than 20 MeV below the experimentally reported $\Xi^* - \Theta^+$ mass difference, emphasizing the need for confirmation of the experimental mass values and placing strong constraints on quark models of the pentaquark structure.

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I. GENERAL ARGUMENTS GIVING BOUNDS ON PENTAQUARK MASSES

There is now a general search for pentaquarks which are related to the Θ^+ [1,2] by changes in flavor of one or more constituents.

The recently observed large mass difference of ≈ 320 MeV between the observed Ξ^* [3,4] state and the Θ^+ places serious constraints on any model. If these two states are members of the same $SU(3)$ antidecuplet [5–9], the mass difference must be due to $SU(3)$ symmetry breaking. We show here using a nearly model-independent variational principle that it is very difficult to fit this mass difference with any simple model [10–13] for symmetry breaking.

We begin with the assumption that the exact wave functions for both the Θ^+ and Ξ^{*-} are eigenfunctions of the same QCD Hamiltonian and that in the $SU(3)$ symmetry limit the masses of the two states are equal and that their wave functions differ only by the interchange of u and s flavors. We can therefore create a variational trial wave function for the Ξ^{*-} by operating on the exact wave function for the Θ^+ with the $SU(3)$ transformation that interchanges the u and s flavors. Although we do not know these exact wave functions and do not know how the QCD Hamiltonian acts on them, we can obtain interesting results for the mass difference by only making simple assumptions about $SU(3)$ symmetry breaking.

The mass obtained with this wave function is an upper bound on the Ξ^{*-} mass by the variational principle. Θ^+ contains one \bar{s} antiquark and 4 light quarks, while Ξ^{*-} contains 2 strange quarks and 3 light (anti)quarks. Since the color field is not changed by this flavor change with the same wave function, the only change in nearly all proposed models is one quark-mass change and a change in the color magnetic energy.

$$M(\Xi^{*-}) \leq M(\Theta^+) + m_s - m_u + \langle \delta V_{hyp}(\bar{s} \rightarrow \bar{u}) \rangle_{\Theta^+} + \langle \delta V_{hyp}(u \rightarrow s) \rangle_{\Theta^+} \quad (1)$$

where $\langle \delta V_{hyp}(\bar{s} \rightarrow \bar{u}) \rangle_{\Theta^+}$ and $\langle \delta V_{hyp}(u \rightarrow s) \rangle_{\Theta^+}$ denote the changes in the color magnetic energy of the exact Θ^+ wave function by the replacements $\bar{s} \rightarrow \bar{u}$ and $u \rightarrow s$.

Although we use the term “color magnetic energy”, motivated by the color hyperfine interaction, the description in terms of the quantities δV_{hyp} is considerably more general and applies to any model in which $SU(3)$ breaking is described by an additive quark mass term and a flavor-dependent-spin dependent two-body quark-quark interaction denoted by V_{hyp} , with its matrix elements determined by fitting ground state baryon masses.

We now estimate these $SU(3)$ breaking effects by using inequalities which are satisfied by most models.

The effective quark mass difference $m_s - m_u$ is given by the Λ -nucleon mass difference [10,14–16]. This gives an upper bound satisfied by nearly all models,

$$m_s - m_u \leq M(\Lambda) - M(N) \quad (2)$$

Note that alternative values of $m_s - m_u$ commonly used are the baryon decuplet mass splittings which give a value much less than the upper bound (2). Using such alternative values of $m_s - m_u$ would therefore make the inequality (1) even more stringent.

Since the color hyperfine interaction is proportional to the effective color magnetic moment, which in turn is inversely proportional to the quark effective mass, the color-magnetic energy of the antiquark can only be lowered by the replacement $\bar{s} \rightarrow \bar{u}$. Therefore

$$\langle \delta V_{hyp}(\bar{s} \rightarrow \bar{u}) \rangle_{\Theta^+} \leq 0 \quad (3)$$

We now estimate the maximum change in the color-magnetic energy by changing the two u -quarks into s -quarks.

The color-magnetic energy between the two u -quarks is repulsive and is lowered by the $u \rightarrow s$ transition. The color-magnetic energy of a ud pair can be raised by the $u \rightarrow s$ transition. We therefore estimate that the maximum change in the color-magnetic energy is obtained by keeping the two u -quarks apart, so that there is no change in their hyperfine energy, and coupling each u -quark with one d -quark to get the maximum change in the hyperfine energy for each. The maximum change to be expected is if it is bound to the d quark in a color antitriplet-spin-zero state, like the ud pair in the Λ , where there is no color-magnetic energy in the interaction with the strange quark. The change in the color-magnetic energy of a ud pair in a color antitriplet-spin-zero state produced by the $u \rightarrow s$ transition is obtainable from the $\Sigma - \Lambda$ mass difference.

We choose a derivation here to minimize model dependence. The ud pairs in the Σ and Λ are both in color-antitriplet s -states and are thus antisymmetric in space and color. They must therefore be symmetric in spin and flavor and have isospins $I = 1; S = 1$ and $I = 0; S = 0$ respectively. In the flavor-symmetry limit, the Σ and Λ masses must be equal, since they are members of the same octet and the contribution of the color magnetic energies to the mass difference of all pairs must cancel. Thus

$$M(\Sigma^-) - M(\Lambda) = [\langle V_{hyp}(ud) \rangle_{(S=1)} - \langle V_{hyp}(sd) \rangle_{(S=1)}] - [\langle V_{hyp}(ud) \rangle_{(S=0)} - \langle V_{hyp}(sd) \rangle_{(S=0)}] \quad (4)$$

This can be rewritten

$$M(\Sigma^-) - M(\Lambda) = (\kappa - 1) \cdot [\langle V_{hyp}(ud) \rangle_{(S=0)} - \langle V_{hyp}(sd) \rangle_{(S=0)}] \quad (5)$$

where

$$\kappa = \frac{\langle V_{hyp}(ud) \rangle_{(S=1)}}{\langle V_{hyp}(ud) \rangle_{(S=0)}} = \frac{\langle V_{hyp}(sd) \rangle_{(S=1)}}{\langle V_{hyp}(sd) \rangle_{(S=0)}} \quad (6)$$

The maximum color-magnetic change occurs if both u quarks have the minimum color magnetic energy,

$$\langle \delta V_{hyp}(u \rightarrow s) \rangle_{\Theta^+} \leq 2 \cdot \langle \delta V_{hyp}(u \rightarrow s) \rangle_{ud(S=0)} = \frac{2}{1 - \kappa} \cdot [M(\Sigma^-) - M(\Lambda)] \quad (7)$$

For a hyperfine interaction proportional to $\vec{\sigma}_1 \cdot \vec{\sigma}_2$, $\kappa = -(1/3)$ and we get the inequality

$$M(\Xi^{*-}) - M(\Theta^+) \leq M(\Lambda) - M(N) + \frac{3}{2} \cdot [M(\Sigma^-) - M(\Lambda)] = 299 \text{ MeV} \quad (8)$$

This is to be compared with the mass difference of ≈ 330 MeV between the reported Ξ^* [3] state and the Θ^+ [1,2]. This slight but significant disagreement emphasizes the need for confirming the experimental mass and places serious constraints on models. This constraint will be seriously violated in any model where the $m_s - m_u$ quark mass difference is less than $M(\Lambda) - M(N)$ or the change in the hyperfine energy including the hyperfine energy of the antiquark neglected in eq. (8) is less than $(3/2) \cdot [M(\Sigma) - M(\Lambda)]$.

An alternative inequality is obtained if the parameter κ is determined from experimental data, rather than the assumption that the interaction is proportional to $\vec{\sigma}_1 \cdot \vec{\sigma}_2$. The Σ^{*-} and Δ^0 decuplet baryons differ by a ($u \rightarrow s$) transition on the Δ^0 . This produces a mass change of one quark mass difference and a change in the color magnetic energy of the two $S = 1$ ud pairs in the Δ^0 . If we assume that the quark mass difference and the color magnetic energy changes are the same in the baryon octet and decuplet, we obtain

$$M(\Sigma^{*-}) - M(\Delta^0) = m_s - m_u + 2\langle\delta V_{hyp}(u \rightarrow s)\rangle_{ud(S=1)} \quad (9)$$

$$M(\Sigma^{*-}) - M(\Delta^0) = m_s - m_u + 2\langle\delta V_{hyp}(u \rightarrow s)\rangle_{ud(S=0)} - 2[M(\Sigma^-) - M(\Lambda)] \quad (10)$$

where we have used eq. (4). Combining eqs. (1), (3) and (10) gives the inequality

$$M(\Xi^{*-}) - M(\Theta^+) \leq M(\Sigma^{*-}) - M(\Delta^0) + 2[M(\Sigma^-) - M(\Lambda)] = 316 \text{ MeV} \quad (11)$$

This uses the assumption that the color magnetic interactions are equal in the octet and decuplet but does not use a theoretical relation between the singlet and triplet splittings to determine the value of κ .

In order to consider analogous bounds for the other members of the Ξ^* multiplet, one needs to take isospin breaking into account. For baryons this is typically of order a few MeV, which is non-negligible compared to the difference between (11) and the $\Xi^* - \Theta^+$ splitting reported by experiments.

Our results apply to any model with a flavor-dependent-spin-dependent two-body quark-quark interaction whose matrix elements satisfy eqs. (3-10) with the parameter κ fixed by fitting the experimental data in eqs. (9) and (11). Any small deviations from these conditions are easily tested by applying our variational principle in any model with a well defined Θ^+ wave function and a well defined prescription for flavor symmetry breaking.

Note that the bounds (10) and (9) were derived with the assumption that the inequality (3) is saturated. This is an extreme assumption which implies no contribution from the color-magnetic interaction of the \bar{s} to the binding of the Θ^+ and it may be unrealistic. It implies that the light quarks are coupled to spin zero and do not interact with the antiquark. However, in any simple model the color-magnetic interaction of the \bar{s} with a quark system coupled to spin zero will polarize the quark system to spin one and reduce the total energy. In this case the constraint (3) will be lowered and the disagreement of with experiment of the variational estimate of $M(\Xi^{*-}) - M(\Theta^+)$ will become more severe. Thus the contribution from the color-magnetic interaction of the \bar{s} should be checked in any model for the Θ^+ and used to strengthen the variational estimate for $M(\Xi^{*-}) - M(\Theta^+)$.

Small mass-dependent effects like the kinetic energy and possible spin-orbit effects have been neglected. Kinetic energy differences are probably included in the effective mass difference, since $M(\Lambda) - M(N)$ includes the difference in the kinetic energies of the s quark in the Λ and the u quark in the proton. If spin-orbit effects pull the Θ^+ down more than the Ξ^{*-} , then the mass gap $M(\Xi^{*-}) - M(\Theta^+)$ can be bigger than the above bounds (8) and (11). This point has been addressed [17] with the interesting result that the $L \cdot S$ contributions for Θ^+ and Ξ^{*-} are expected to be very similar and should only have a small affect on the bounds.

NOTE ADDED – APRIL 2004

Recently two experiments announced null results in search for Ξ^{*-} [18,19]. Since the production mechanism is yet unknown, at this time it is not clear whether or not these results are compatible with those of Ref. [3].

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